Axially asymmetric fermion scattering off electroweak phase transition bubble walls with hypermagnetic fields

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We show that in the presence of large scale primordial hypermagnetic fields it is possible to generate an axial asymmetry for a first order electroweak phase transition. This happens during the reflection and transmission of fermions off the true vacuum bubbles, due to the chiral nature of the fermion coupling with the background field in the symmetric phase. We derive and solve the Dirac equation for such fermions and compute the reflection and transmission coefficients for the case when these fermions move from the symmetric to the symmetry broken phase. We also comment on the possible implications of such axial charge segregation processes for baryon number generation.

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I. INTRODUCTION

One of the most challenging problems for particle physics as applied to cosmology is the explanation of the observed excess of baryons over antibaryons in the universe. For this purpose, a theory has to meet the three well-known Sakharov conditions [1]: namely; (1) the existence of interactions that violate baryon number; (2) C and CP violation and (3) departure from thermal equilibrium. The above conditions are met in the standard model (SM) provided the electroweak phase transition (EWPT) is of first order. This has raised the interesting possibility that the cosmological phase transition that gave rise to the mass of particles, which took place at temperatures of order 100 GeV, could also explain the generation of baryon number. Consequently, a great deal of effort has been devoted to exploring this possibility [2].

Nowadays, the consensus is that the minimal SM, as such, cannot explain the observed baryon number. The reason is that the EWPT turns out to be only too weakly first order which in turn implies that any baryon asymmetry generated at the phase transition was erased by the same mechanism that produced it, i.e., sphaleron induced processes [3]. Moreover, the amount of CP violation coming from the Cabibbo-Kobayashi-Maskawa (CKM) matrix alone cannot account by itself for the observed asymmetry, given that its effect shows up in the coupling of the Higgs boson with fermions at a high perturbative order [4], producing a baryon to entropy ratio at least ten orders of magnitude smaller than the observed one.

Nevertheless, it has been recently pointed out that, provided a source of enough CP violation exists, the above scenario could significantly change in the presence of large-scale primordial magnetic fields [5–7] (see however Ref. [8]), which can be responsible for a stronger first-order EWPT. This situation is analogous to the case of a type I superconductor in which the presence of an external magnetic field modifies the order of the phase transition due to the Meissner effect. Although the nature of these fields is a subject of current research, their existence prior to the EWPT epoch cannot certainly be ruled out [9].

Magnetic fields have been observed in many astrophysical objects. Estimation of their strengths require independent knowledge of the local electron density and the spatial structure of the field. Both quantities are reasonably well known for our galaxy, where the average field strength has been measured to be between 3 and 4 μG; moreover, various spiral galaxies in our neighborhood present similar magnetic field strengths [10]. At larger scales, only model dependent upper limits can be established and these are also in the few microgauss range. Magnetic fields at the microgauss level have been observed as well in high-redshift objects. In the intergalactic medium, adopting some reasonable values for the magnetic coherence length, the upper bound of 10^{-9} G has been estimated [9]. The origin of these fields is currently unknown but it is widely believed that, in order to produce them, two ingredients are needed: a mechanism for creating the seed fields and a process for amplifying both their amplitude and their coherence scale.

Generation of the seed field (magnetogenesis) may either be primordial or be produced during the process of structure formation. In the early universe, which is the case of interest here, there are a number of proposed mechanisms that could possibly generate large-scale primordial fields. Among the best suited are first order phase transitions [11,12], which provide favorable conditions such as charge separation, turbulence and departure from equilibrium. In particular, bubble wall collisions produce phase gradients of a complex order parameter that act as a source for gauge fields [13]. When interested in larger coherence scales, a plausible scenario is inflation, where superhorizon scale fields are generated through the amplification of quantum fluctuations of the gauge fields. This process however, needs, a mechanism for breaking conformal invariance of the electromagnetic field [14].

The most promising way to distinguish between primordial and protogalactic fields is through the search for their imprint on the cosmic microwave background radiation (CMBR). Temperature anisotropies from Cosmic Background Explorer (COBE) results place an upper bound $B_0 \sim 10^{-9}$ G for homogeneous fields (B_0 refers to the intensity that the field would have today under the assumption of adiabatic decay due to the Hubble expansion) [15]. In the case of inhomogeneous fields their effect must be searched for in the Doppler peaks [16] and in the polarization of the CMBR [17]. The future CMBR satellite missions Microwave Anisotropy Probe (MAP) and Planck may reach the required sensitivity for the detection of these last signals.

Independently of their origin, primordial fields could have had some influence on physical processes which occurred in the early universe, like big-bang nucleosynthesis and electroweak baryogenesis.

Recall that for temperatures above the EWPT, the $SU(2)\times U(1)_Y$ symmetry is restored and the propagating, non-screened vector modes that represent a magnetic field correspond to the $U(1)_Y$ group instead of to the $U(1)_{em}$ group, and are therefore properly called *hypermagnetic* fields.

In this paper we use a simple model to show that the presence of such fields also provides a mechanism, working in the same manner as the existence of additional CP violation within the SM, to produce an axial charge segregation during the EWPT. This happens in the scattering of fermions off the true vacuum bubbles nucleated during the phase transition and is a consequence of the chiral nature of the fermion coupling to hypermagnetic fields in the symmetric phase.

The outline of this work is as follows. In Sec. II, we write the Dirac equation for the left- and right-handed chirality modes propagating in a background hypermagnetic field during the EWPT. In Sec. III, we find the solution and discuss its properties. In Sec. IV, we use this solution to compute reflection and transmission probabilities. We show that these probabilities differ for the two distinct chirality modes. Finally in Sec. V, we conclude by looking at the possible implications of such axially asymmetric fermion reflection and transmission.

II. DIRAC EQUATION FOR FERMIONS MOVING IN A BACKGROUND HYPERMAGNETIC FIELD

In a first order phase transition, the conversion from one phase to another happens through nucleation. The region separating the two phases is called the wall. During the EWPT, the properties of the wall depend on the effective, finite temperature Higgs potential. Under the assumption that the wall is thin and that the phase transition happens when the energy densities of both phases are degenerate, it is possible to find a one-dimensional analytical solution for the Higgs field ϕ called the *kink*. This is given by

$$\phi(z) \sim 1 + \tanh(z/\lambda), \tag{1}$$

where z is the coordinate along the direction of the phase change and λ is the width of the wall. When scattering is not affected by diffusion, the problem of fermion reflection and transmission through the wall can be cast in terms of solving the Dirac equation with a position dependent fermion mass, proportional to the Higgs field [18]. Let us further simplify the problem by considering the limit when the width of the wall approaches zero. In this case, the kink solution becomes a step function, $\Theta(z)$, and consequently the expression for the particle's mass becomes

$$m(z) = m_0 \Theta(z). \tag{2}$$

In terms of Eq. (2), we can see that $z \leqslant 0$ represents the region outside the bubble, that is, the region in the symmetric phase where particles are massless. Conversely, for $z \geqslant 0$, the system is inside the bubble, that is, in the broken phase, and the particles have acquired a finite mass m_0.

In the presence of an external magnetic field, we need to consider that fermion modes couple differently to the field in the broken and the symmetric phases. We start the analysis looking at the unbroken phase.

For $z \leqslant 0$, the coupling is chiral. Let

$$\Psi_R = \frac{1}{2}(1+\gamma_5)\Psi$$

$$\Psi_L = \frac{1}{2}(1-\gamma_5)\Psi \tag{3}$$

represent, as usual, the right- and left-handed chirality modes for the spinor Ψ, respectively. Then, the equations of motion for these modes, as derived from the electroweak interaction Lagrangian, are

$$\left(i\partial\!\!\!/ - \frac{y_L}{2} g' A\!\!\!/ \right)\Psi_L - m(z)\Psi_R = 0$$

$$\left(i\partial\!\!\!/ - \frac{y_R}{2} g' A\!\!\!/ \right)\Psi_R - m(z)\Psi_L = 0, \tag{4}$$

where $y_{R,L}$ are the right- and left-handed hypercharges corresponding to the given fermion, respectively, g' is the $U(1)_Y$ coupling constant and we take $A^\mu = (0,\mathbf{A})$ representing a, not yet specified, four-vector potential having nonzero components only for its spatial part, in the rest frame of the wall.

The set of Eqs. (4) can be written as a single equation for the spinor $\Psi = \Psi_R + \Psi_L$ by adding up the former equations:

$$\left\{ i\partial\!\!\!/ - A\!\!\!/ \left[\frac{y_R}{4} g'(1+\gamma_5) + \frac{y_L}{4} g'(1-\gamma_5) \right] - m(z) \right\}\Psi = 0. \tag{5}$$

Hereafter, we explicitly work in the chiral representation of the gamma matrices where

$$\gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}, \quad \boldsymbol{\gamma} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}. \tag{6}$$

Within this representation, we can write Eq. (5) as

$$\{ i\partial\!\!\!/ - \mathcal{G} A_\mu \gamma^\mu - m(z) \}\Psi = 0, \tag{7}$$

where we have introduced the matrix

$$\mathcal{G}=\begin{pmatrix} \frac{y_L}{2}g'I & 0 \\ 0 & \frac{y_R}{2}g'I \end{pmatrix}. \tag{8}$$

We now look at the corresponding equation in the broken-symmetry phase. For $z\geqslant 0$ the coupling of the fermion with the external field is through the electric charge e and thus the equation of motion is simply the Dirac equation describing an electrically charged fermion in a background magnetic field, namely,

$$\{i\partial\!\!\!/-eA_\mu\gamma^\mu-m(z)\}\Psi=0. \tag{9}$$

In the following section, we explicitly construct the solutions to Eqs. (7) and (9) with a constant magnetic field, requiring that these match at the interface $z=0$.

III. SOLVING THE DIRAC EQUATION

Let us first find the solution to Eq. (7), namely, for fermions moving in the symmetric phase, $z\leqslant 0$. For this purpose, we look for a solution of the form

$$\Psi=\{i\partial\!\!\!/-A_\mu\gamma^\mu\mathcal{G}+m(z)\}\Phi. \tag{10}$$

Inserting this expression into Eq. (7), we obtain

$$\{-\partial^2-i\mathcal{G}\partial^\mu A_\mu-\tfrac{1}{2}\sigma^{\mu\nu}\mathcal{G}F_{\mu\nu}-2i\mathcal{G}A_\mu\partial^\mu+\mathcal{G}^2A_\mu A^\mu +i\gamma^\mu\partial_\mu m(z)\}\Phi=0, \tag{11}$$

where, as usual,

$$\sigma^{\mu\nu}=\frac{i}{2}[\gamma^\mu,\gamma^\nu]$$

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu. \tag{12}$$

For definiteness, let us consider a constant magnetic field $\mathbf{B}=B\hat{z}$ pointing along the $\hat{z}$ direction. In this case, the vector potential $\mathbf{A}$ can only have components perpendicular to $\hat{z}$ and the solution to Eq. (11) factorizes as [19]

$$\Phi(t,\boldsymbol{x})=\zeta(x,y)\Phi(t,z). \tag{13}$$

We concentrate on the solution describing the motion of fermions perpendicular to the wall, i.e., along the $\hat{z}$ axis, and, furthermore, look for stationary states: namely,

$$\Phi(t,z)=e^{-iEt}\Phi(z). \tag{14}$$

Therefore, working in the Lorentz gauge, $\partial^\mu A_\mu=0$, Eq. (11) becomes

$$\left\{\frac{d^2}{dz^2}+i\gamma^3\frac{dm(z)}{dz}+E^2+iB\mathcal{G}\gamma^1\gamma^2\right\}\Phi(z)=0. \tag{15}$$

Notice that Eqs. (11) and (15) have the appropriate limit when $y_R=y_L=e$, corresponding to the description of fermions coupled with their electric charge to a background magnetic field [19].

We now expand $\Phi(z)$ in terms of the eigenspinors $u^s_\pm$ $(s=\mathbf{1},\mathbf{2})$ of γ^3,

$$u^1_\pm=\begin{pmatrix}1\\0\\\pm i\\0\end{pmatrix},\quad u^2_\pm=\begin{pmatrix}0\\1\\0\\\mp i\end{pmatrix}. \tag{16}$$

These spinors have the properties

$$\begin{aligned}\gamma^3u^{1,2}_\pm&=\pm iu^{1,2}_\pm\\ \gamma^0u^1_\pm&=\mp iu^1_\mp\\ \gamma^0u^2_\pm&=\pm iu^2_\mp\\ \gamma^1\gamma^2u^1_\pm&=-iu^1_\pm\\ \gamma^1\gamma^2u^2_\pm&=+iu^2_\pm\\ \gamma_5u^{1,2}_\pm&=u^{1,2}_\mp.\end{aligned} \tag{17}$$

Writing

$$\Phi(z)=\phi^1_+(z)u^1_++\phi^1_-(z)u^1_-+\phi^2_+(z)u^2_++\phi^2_-(z)u^2_- \tag{18}$$

and inserting this expression into Eq. (15), we obtain

$$\begin{aligned}&\left[\frac{d^2}{dz^2}+E^2+g'\frac{(y_L+y_R)}{4}B\right]\phi^1_+(z)+g'\frac{(y_L-y_R)}{4}B\phi^1_-(z)\\&\quad=m_0\delta(z)\phi^1_+(z)\\&\left[\frac{d^2}{dz^2}+E^2+g'\frac{(y_L+y_R)}{4}B\right]\phi^1_-(z)\\&\quad+g'\frac{(y_L-y_R)}{4}B\phi^1_+(z)\\&\quad=-m_0\delta(z)\phi^1_-(z),\end{aligned} \tag{19}$$

and

$$\begin{aligned}&\left[\frac{d^2}{dz^2}+E^2-g'\frac{(y_L+y_R)}{4}B\right]\phi^2_+(z)-g'\frac{(y_L-y_R)}{4}B\phi^2_-(z)\\&\quad=m_0\delta(z)\phi^2_+(z)\\&\left[\frac{d^2}{dz^2}+E^2-g'\frac{(y_L+y_R)}{4}B\right]\phi^2_-(z)-g'\frac{(y_L-y_R)}{4}B\phi^2_+(z)\\&\quad=-m_0\delta(z)\phi^2_-(z).\end{aligned} \tag{20}$$

Equations (19) and (20) each represent a set of two coupled second-order differential equations. The second set is obtained from the first one by changing B to $-B$. Consequently, Eqs. (19) and the corresponding functions and spinors with $s=1$ describe the motion of the spin components parallel to the magnetic field whereas Eqs. (20) and the functions and spinors with $s=2$ describe the motion of the spin components antiparallel to the magnetic field. Notice that in the limit when $y_R=y_L=e$, each set of equations decouples as is the case when describing the interaction of fermions with the magnetic field through their electric charge.

Let us focus on the set of Eqs. (19), since, as we have pointed out, the solutions to Eqs. (20) are obtained from those to Eqs. (19) by changing B to $-B$.

To solve Eqs. (19), we look for the scattering states appropriate to describe the motion of fermions in the symmetric phase. For our purposes, these are fermions incident toward and reflected from the wall. There are two types of such solutions; those coupled with y_L and those coupled with y_R. For an incident wave coupled with y_L (y_R), the fact that the differential Eqs. (19) mix up the solutions means that the reflected wave will also include a component coupled with y_R (y_L). Let us classify the solutions according to the type of wave that is incident toward the wall. For an incident wave coupled with y_L, which we call type (a), the solutions $\phi_{\pm}^{1(a)}(z)$ are

$$\phi_{\pm}^{1(a)}(z)=e^{i\alpha_1^L z}-\frac{m_0^2}{4\alpha_1^L\alpha_1^R+m_0^2}e^{i\alpha_1^L|z|}\mp\frac{2im_0\alpha_1^L}{4\alpha_1^L\alpha_1^R+m_0^2}e^{i\alpha_1^R|z|}, \tag{21}$$

whereas, for an incident wave coupled with y_R, which we call type (b), the solutions $\phi_{\pm}^{1(b)}(z)$ are

$$\phi_{\pm}^{1(b)}(z)=\pm e^{i\alpha_1^R z}-\frac{2im_0\alpha_1^R}{4\alpha_1^L\alpha_1^R+m_0^2}e^{i\alpha_1^L|z|}\mp\frac{m_0^2}{4\alpha_1^L\alpha_1^R+m_0^2}e^{i\alpha_1^R|z|}, \tag{22}$$

where we use the notation

$$\alpha_1^{R,L}=\sqrt{E^2+\frac{y_{R,L}g'}{2}B}. \tag{23}$$

It is a straightforward exercise to verify that the functions $\phi_{\pm}^{1(a,b)}(z)$ given by Eqs. (21) and (22) indeed satisfy the system of Eqs. (19).

The corresponding fermion wave functions are given in terms of Eq. (10). Taking $E>0$ and in the approximation where we look only at the part of the wave function that describes motion perpendicular to the wall, we obtain, for solutions of type (a),

$$\Psi_{\rm inc}^{(a)}(z)=-i(\alpha_1^L-E)(u_+^1-u_-^1)e^{i\alpha_1^L z}-i(\alpha_2^L+E)\times(u_+^2-u_-^2)e^{i\alpha_2^L z} \tag{24}$$

$$\begin{aligned}\Psi_{\rm ref}^{(a)}(z)=&\frac{-im_0}{4\alpha_1^L\alpha_1^R+m_0^2}\{m_0(\alpha_1^L+E)(u_+^1-u_-^1)e^{-i\alpha_1^L z}\\&+2i\alpha_1^L(\alpha_1^R-E)(u_+^1+u_-^1)e^{-i\alpha_1^R z}\}\\&-\frac{im_0}{4\alpha_2^L\alpha_2^R+m_0^2}\{m_0(\alpha_2^L-E)(u_+^2\\&-u_-^2)e^{-i\alpha_2^L z}+2i\alpha_2^L(\alpha_2^R+E)(u_+^2\\&+u_-^2)e^{-i\alpha_2^R z}\},\end{aligned}$$

whereas, for solutions of type (b),

$$\Psi_{\rm inc}^{(b)}(z)=-i(\alpha_1^R+E)(u_+^1+u_-^1)e^{i\alpha_1^R z}-i(\alpha_2^R-E)\times(u_+^2+u_-^2)e^{i\alpha_2^R z}$$

$$\begin{aligned}\Psi_{\rm ref}^{(b)}(z)=&\frac{-im_0}{4\alpha_1^L\alpha_1^R+m_0^2}\{m_0(\alpha_1^R-E)(u_+^1+u_-^1)e^{-i\alpha_1^R z}\\&+2i\alpha_1^R(\alpha_1^L+E)(u_+^1-u_-^1)e^{-i\alpha_1^L z}\}\\&-\frac{im_0}{4\alpha_2^L\alpha_2^R+m_0^2}\{m_0(\alpha_2^R+E)\\&\times(u_+^2+u_-^2)e^{-i\alpha_2^R z}+2i\alpha_2^R(\alpha_2^L-E)\\&\times(u_+^2-u_-^2)e^{-i\alpha_2^L z}\},\end{aligned} \tag{25}$$

where, in analogy with Eq. (23), we define

$$\alpha_2^{R,L}=\sqrt{E^2-\frac{y_{R,L}g'}{2}B}. \tag{26}$$

We now turn to finding the solution to Eq. (9), namely, for fermions moving in the broken phase, $z\geqslant 0$. This time, we look for a solution of the form

$$\Psi=\{i\partial\!\!\!/-eA_\mu\gamma^\mu+m(z)\}\Phi. \tag{27}$$

By a procedure similar to that leading to Eqs. (19) and (20), the corresponding equations for the functions $\phi_{\pm}^{1,2}(z)$ in this region become

$$\left[\frac{d^2}{dz^2}+E^2-m_0^2+eB\right]\phi_{\pm}^1(z)=\pm m_0\delta(z)\phi_{\pm}^1(z)$$

$$\left[\frac{d^2}{dz^2}+E^2-m_0^2-eB\right]\phi_{\pm}^2(z)=\pm m_0\delta(z)\phi_{\pm}^2(z). \tag{28}$$

As expected, when the coupling of the fermion with the external magnetic field is through its electric charge, the equations describing the behavior of the functions $\phi_{\pm}^{1,2}(z)$ decouple. For our purposes, we look for the scattering states appropriate for the description of transmitted waves. These are

$$\phi_{\pm}^{1,2}(z)=e^{i\alpha_{1,2}z}\mp\frac{im_0}{2\alpha_{1,2}\pm im_0}e^{i\alpha_{1,2}|z|}, \tag{29}$$

where we use the notation

$$\alpha_1=\sqrt{E^2-m_0^2+eB}$$
$$\alpha_2=\sqrt{E^2-m_0^2-eB}. \tag{30}$$

It is also a straightforward exercise to verify that Eq. (29) indeed satisfies the set of Eqs. (28). The fermion wave function is obtained from Eq. (27). Also, for $E>0$ and in the approximation where we look only at the part describing the motion of particles along $\hat{z}$, and furthermore imposing continuity of the fermion wave function at $z=0$, we obtain for solutions of type (a)

$$\begin{aligned}\Psi_{\rm tra}^{(a)}(z)=&\frac{2\alpha_1^L}{4\alpha_1^L\alpha_1^R+m_0^2}\{m_0(\alpha_1^R-E)(u_+^1+u_-^1)\\&-i[2\alpha_1^R(\alpha_1^L-E)+m_0^2](u_+^1-u_-^1)\}e^{i\alpha_1 z}\\&+\frac{2\alpha_2^L}{4\alpha_2^L\alpha_2^R+m_0^2}\{m_0(\alpha_2^R+E)(u_+^2+u_-^2)\\&-i[2\alpha_2^R(\alpha_2^L+E)+m_0^2](u_+^2-u_-^2)\}e^{i\alpha_2 z},\end{aligned} \tag{31}$$

and for solutions of type (b)

$$\begin{aligned}\Psi_{\rm tra}^{(b)}(z)=&\frac{2\alpha_1^R}{4\alpha_1^L\alpha_1^R+m_0^2}\{m_0(\alpha_1^L+E)(u_+^1-u_-^1)\\&-i[2\alpha_1^L(\alpha_1^R+E)+m_0^2](u_+^1+u_-^1)\}e^{i\alpha_1 z}\\&+\frac{2\alpha_2^R}{4\alpha_2^L\alpha_2^R+m_0^2}\{m_0(\alpha_2^L-E)(u_+^2-u_-^2)\\&-i[2\alpha_2^L(\alpha_2^R-E)+m_0^2](u_+^2+u_-^2)\}e^{i\alpha_2 z}.\end{aligned} \tag{32}$$

Recall that in the absence of the hypermagnetic field, the eigenvalues of the chirality and the helicity operators (χ and h, respectively) are the same. The presence of the external field lifts such degeneracy and the eigenstates of chirality no longer have a definite helicity. Nevertheless, it is easy to check that for field strengths eB smaller than m_0^2, the component with h that would correspond to a given χ in the absence of the external field dominates over the rest of the components. For $E>0$, this means that, to a good approximation, left- (right-)handed particles are transmitted as such (in both chirality and helicity) but become right- (left-)handed (in both chirality and helicity) upon reflection. In these cases and to a good approximation, the quantum number conserved during scattering off the wall is the ratio $\chi/h=1$. It can also be shown [20] that to a good approximation, for $E<0$, the corresponding conserved quantum number is $\chi/h=-1$.

IV. REFLECTION AND TRANSMISSION PROBABILITIES

The fact that the amplitudes in Eqs. (25) and (32) are not the same as those in Eqs. (24) and (31) means that an axial asymmetry is built during the scattering of fermions off the wall. To quantify the asymmetry, we need to compute the corresponding reflection and transmission coefficients. These are built from the reflected, transmitted and incident currents of each type. Recall that for a given spinor wave function Ψ, the current normal to the wall is given by

$$J=\Psi^\dagger\gamma^0\gamma^3\Psi. \tag{33}$$

As can be seen from Eqs. (25),(32) and (24),(31), an incident wave with a given chirality [left-handed for waves of type (a), right-handed for waves of type (b)], contains, upon reflection and transmission, both kinds of chirality mode. For waves of type (a), the corresponding currents are

$$J_{\rm inc}^{(a)}=4\{(\alpha_2^L+E)^2-(\alpha_1^L-E)^2\}$$
$$J_{\rm ref}^{(a)}=J_{\rm ref}^{(a)R}+J_{\rm ref}^{(a)L}$$

where

$$J_{\rm ref}^{(a)R}=-4m_0^2\left\{\left(\frac{2\alpha_2^L}{4\alpha_2^L\alpha_2^R+m_0^2}\right)^2(\alpha_2^R+E)^2-\left(\frac{2\alpha_1^L}{4\alpha_1^L\alpha_1^R+m_0^2}\right)^2(\alpha_1^R-E)^2\right\}$$

$$J_{\rm ref}^{(a)L}=-4m_0^2\left\{\left(\frac{m_0}{4\alpha_1^L\alpha_1^R+m_0^2}\right)^2(\alpha_1^L+E)^2-\left(\frac{m_0}{4\alpha_2^L\alpha_2^R+m_0^2}\right)^2(\alpha_2^L-E)^2\right\}$$

and

$$J_{\rm tra}^{(a)}=J_{\rm tra}^{(a)R}+J_{\rm tra}^{(a)L}$$

where

$$J_{\rm tra}^{(a)R}=16\left\{\left(\frac{m_0\alpha_1^L(\alpha_1^R-E)}{4\alpha_1^L\alpha_1^R+m_0^2}\right)^2-\left(\frac{m_0\alpha_2^L(\alpha_2^R+E)}{4\alpha_2^L\alpha_2^R+m_0^2}\right)^2\right\}$$

$$J_{\rm tra}^{(a)L}=16\left\{\left(\frac{2\alpha_2^L\alpha_2^R(\alpha_2^L+E)+m_0^2\alpha_2^L}{4\alpha_2^L\alpha_2^R+m_0^2}\right)^2-\left(\frac{2\alpha_1^L\alpha_1^R(\alpha_1^L-E)+m_0^2\alpha_1^L}{4\alpha_1^L\alpha_1^R+m_0^2}\right)^2\right\}, \tag{34}$$

whereas for waves of type (b), the corresponding currents are

$$J_{\rm inc}^{(b)}=4\{(\alpha_1^R+E)^2-(\alpha_2^R-E)^2\}$$
$$J_{\rm ref}^{(b)}=J_{\rm ref}^{(b)R}+J_{\rm ref}^{(b)L}$$

where

$$J_{\rm ref}^{(b)R}=-4m_0^2\left\{\left(\frac{m_0}{4\alpha_2^L\alpha_2^R+m_0^2}\right)^2(\alpha_2^R+E)^2-\left(\frac{m_0}{4\alpha_1^L\alpha_1^R+m_0^2}\right)^2(\alpha_1^R-E)^2\right\}$$

$$J_{\rm ref}^{(b)L}=-4m_0^2\left\{\left(\frac{2\alpha_1^R}{4\alpha_1^L\alpha_1^R+m_0^2}\right)^2(\alpha_1^L+E)^2-\left(\frac{2\alpha_2^R}{4\alpha_2^L\alpha_2^R+m_0^2}\right)^2(\alpha_2^L-E)^2\right\}$$

and

$$J_{\rm tra}^{(b)}=J_{\rm tra}^{(b)R}+J_{\rm tra}^{(b)L}$$

where

$$J_{\rm tra}^{(b)R}=16\left\{\left(\frac{2\alpha_1^L\alpha_1^R(\alpha_1^R+E)+m_0^2\alpha_1^R}{4\alpha_1^L\alpha_1^R+m_0^2}\right)^2-\left(\frac{2\alpha_2^L\alpha_2^R(\alpha_2^R-E)+m_0^2\alpha_2^R}{4\alpha_2^L\alpha_2^R+m_0^2}\right)^2\right\}$$

$$J_{\rm tra}^{(b)L}=16\left\{\left(\frac{m_0\alpha_2^R(\alpha_2^L-E)}{4\alpha_2^L\alpha_2^R+m_0^2}\right)^2-\left(\frac{m_0\alpha_1^R(\alpha_1^L+E)}{4\alpha_1^L\alpha_1^R+m_0^2}\right)^2\right\}. \tag{35}$$

The reflection and transmission coefficients are given as the ratios of the reflected and transmitted currents to the incident one, respectively, projected along a unit vector normal to the wall,

$$R_{L\to L}=-J_{\rm ref}^{(a)L}/J_{\rm inc}^{(a)}$$

$$R_{L\to R}=-J_{\rm ref}^{(a)R}/J_{\rm inc}^{(a)}$$

$$T_{L\to L}=J_{\rm tra}^{(a)L}/J_{\rm inc}^{(a)}$$

$$T_{L\to R}=J_{\rm tra}^{(a)R}/J_{\rm inc}^{(a)}. \tag{36}$$

Equations (36) represent the probabilities that a left-handed incident particle bounces off the wall as a left- or a right-handed particle or is transmitted through the wall as a left- or a right-handed particle, respectively. The corresponding probabilities for the axially conjugate processes are

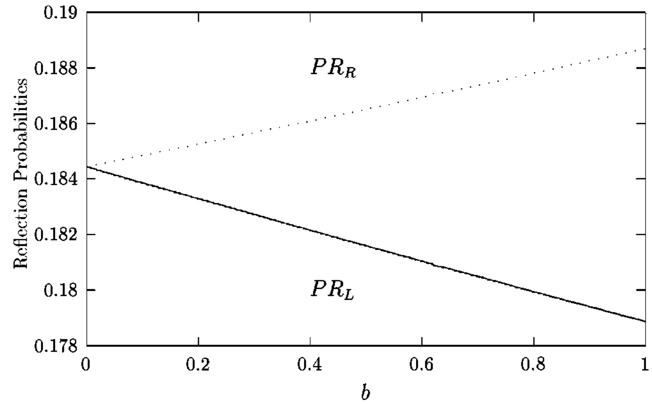

FIG. 1. Probabilities PR_L and PR_R as functions of the magnetic field parametrized as $B=bT^2$ for $T=100$ GeV, $E=184$ GeV and a top quark with a mass $m_0=175$ GeV, $y_R=4/3$, $y_L=1/3$. The value for the $U(1)_Y$ coupling constant is taken as $g'=0.344$, corresponding to the EWPT epoch.

$$R_{R\to L}=-J_{\rm ref}^{(b)L}/J_{\rm inc}^{(b)}$$

$$R_{R\to R}=-J_{\rm ref}^{(b)R}/J_{\rm inc}^{(b)}$$

$$T_{R\to L}=J_{\rm tra}^{(b)L}/J_{\rm inc}^{(b)}$$

$$T_{R\to R}=J_{\rm tra}^{(b)R}/J_{\rm inc}^{(b)}. \tag{37}$$

Therefore, the probabilities for finding a left- or a right-handed particle in the symmetric phase after reflection, PR_L and PR_R, are given, respectively, by

$$PR_L=R_{L\to L}+R_{R\to L}$$

$$PR_R=R_{L\to R}+R_{R\to R}, \tag{38}$$

whereas the probabilities for finding a left- or a right-handed particle in the symmetry broken phase after transmission, PT_L and PT_R, are given, respectively, by

$$PT_L=T_{L\to L}+T_{R\to L}$$

$$PT_R=T_{L\to R}+T_{R\to R}. \tag{39}$$

Figure 1 shows the probabilities PR_L and PR_R as functions of the magnetic field parametrized as $B=bT^2$ for a temperature $T=100$ GeV, a fixed $E=184$ GeV and for a fermion taken as the top quark with a mass $m_0=175$ GeV, $y_R=4/3$, $y_L=1/3$ and for a value of $g'=0.344$, as appropriate for the EWPT epoch. Notice that when $b\to 0$, these probabilities approach each other, and that the difference grows with increasing field strength. Also, in order to be able to safely neglect the contribution from the negative energy solutions, we are bound to consider not too large values of the parameter b. For the purposes of this work, we take a maximum value of $b=1$, which for the values of T and m_0 considered, amounts for a maximum fraction of the magnetic energy to the particle's rest mass of order $\sqrt{eB}/m_0\sim 0.3$.

Figure 2 shows the reflection and transmission probabilities as a function of the particle's energy E. Figure 2(a)

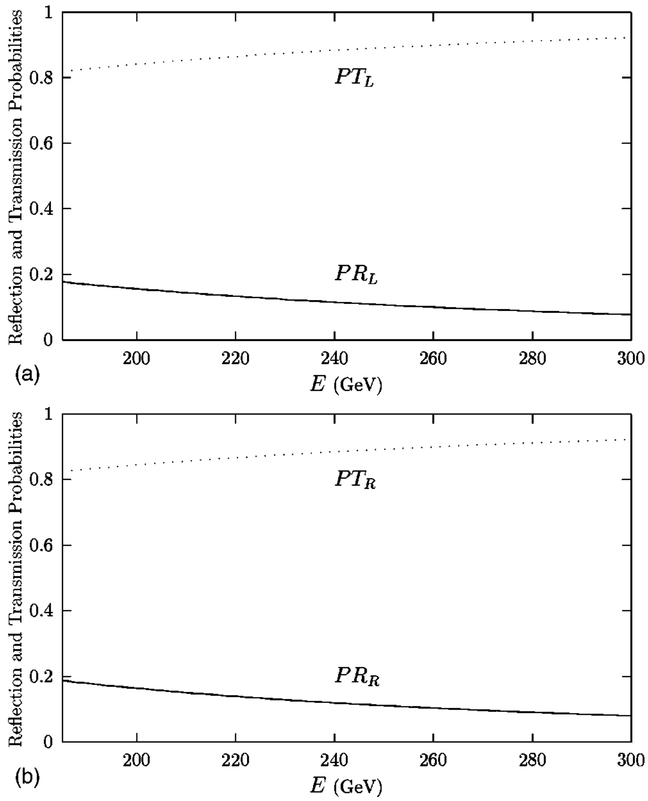

FIG. 2. Reflection and transmission probabilities as a function of the particle's energy E. (a) (upper panel) shows the probabilities PR_L and PT_L. (b) (lower panel) shows the probabilities PR_R and PT_R. In both cases, the strength of the magnetic field is taken with $b=1$ and $T=100$ GeV. Also $m_0=175$ GeV, $y_R=4/3$, $y_L=1/3$, corresponding to a top quark.

shows the probabilities PR_L and PT_L and Fig. 2(b) the probabilities PR_R and PT_R for $b=1$. As before, the parameters considered correspond to a top quark. Since the solutions in Eqs. (31) and (32) are computed assuming that the transmitted waves are not exponentially damped, the energy has to be taken such that the parameters $\alpha_{1,2}$ in Eqs. (30) are real, which in turn implies that $E \geqslant \sqrt{m_0^2+eB}$. It can be numerically checked that $PR_L+PT_L=PR_R+PT_R=1$ to within a maximum deviation of one part in one thousand. The fact that these probabilities add up to 1 is equivalent to current conservation:

$$J_{\rm tra}^{(i)} - J_{\rm ref}^{(i)} = J_{\rm inc}^{(i)} \tag{40}$$

$(i=a,b)$, as a consequence of the equality of the currents:

$$J_{\rm tra}^{(a)R} = J_{\rm ref}^{(a)R}$$
$$J_{\rm tra}^{(b)L} = J_{\rm ref}^{(b)L}, \tag{41}$$

as can be checked from the sets of Eqs. (34) and (35).

V. CONCLUSIONS AND OUTLOOK

In this paper we have derived and solved the Dirac equation for fermions scattering off a first order EWPT bubble wall in the presence of a magnetic field directed along the fermion direction of motion. In the symmetric phase, the fermions couple chirally to the magnetic field, which receives the name of *hypermagnetic*, given that it belongs to the $U(1)_Y$ group. We have shown that the chiral nature of this coupling implies that it is possible to build an axial asymmetry during the scattering of fermions off the wall. We have computed reflection and transmission coefficients, showing explicitly that they differ for left- and right-handed incident particles from the symmetric phase.

Recall that under the very general assumptions of CPT invariance, together with conservation of unitarity, which are satisfied in the present analysis, the total axial asymmetry (which includes contributions from both particles and antiparticles) is quantified in terms of the particle (axial) asymmetry. Let ρ_i represent the number density for species i. The net densities in left-handed and right-handed axial charges are obtained by taking the differences $\rho_L-\rho_{\bar{L}}$ and $\rho_R-\rho_{\bar{R}}$, respectively. It is straightforward to show [21] that CPT invariance and unitarity imply that the above net densities are given by

$$\rho_L-\rho_{\bar{L}}=(f^s-f^b)(PR_L-PR_R)$$
$$\rho_R-\rho_{\bar{R}}=(f^s-f^b)(PR_R-PR_L), \tag{42}$$

where f^s and f^b are the statistical distributions for particles or antiparticles (since the chemical potentials are assumed to be zero or small compared to the temperature, these distributions are the same for particles or antiparticles) in the symmetric and the symmetry-broken phases, respectively. From Eq. (42), the asymmetry in the axial charge density is finally given by

$$(\rho_L-\rho_{\bar{L}})-(\rho_R-\rho_{\bar{R}})=2(f^s-f^b)(PR_L-PR_R). \tag{43}$$

This asymmetry in the axial charge, built on either side of the wall, is dissociated from nonconserving baryon number processes and can subsequently be converted to baryon number in the unbroken phase where sphaleron induced transitions are taking place with a large rate. This mechanism receives the name of *nonlocal baryogenesis* [21–24] and, in the absence of the external field, it can be realized only in extensions of the SM where a source of CP violation is introduced *ad hoc* into a complex, space-dependent phase of the Higgs field during the development of the EWPT [25].

Due to the sphaleron dipole moment, another consequence of the existence of an external magnetic field is the lowering of the barrier between topologically inequivalent vacua [26]. This effect acts in such a way that any baryon asymmetry generated by the building of an axial charge during the asymmetric reflection of fermions into the unbroken phase, in the presence of a magnetic field, stands little chance of surviving in the broken phase. Nonetheless, if such primordial fields indeed existed during the EWPT epoch and the phase transition was first order, as is the case, for instance, in minimal extensions of the SM, the mechanism advocated in this work has to be considered as acting in the same manner as a source of CP violation that can have important consequences for the generation of a baryon number. These matters will be the subject of an upcoming work [20].

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